VEHICLE DISPATCHING PLAN FOR MINIMIZING PASSENGER WAITING TIME IN A CORRIDOR WITH BUSES OF DIFFERENT SIZES: MODEL FORMULATION AND SOLUTION APPROACHES.

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MOTIVATION

A vehicle dispatching plan was developed in order to:

- Minimize passenger waiting time
- Minimize denied boardings
- More efficient utilization of available resources in terms of vehicle capacity especially during peak period/(improving capacity utilization
- To meet the passenger demand

PREVIOUS WORK

With the work by dell'Olio, Ibeas and Ruisánchez (2012), the following observations were made:

An optimization model with constraints on bus capacity to optimize bus size and headway was constructed

Experiments on homogenous fleets made up of buses of the same size were conducted,

Experiments on heterogeneous fleets composed of different bus sizes were conducted

The findings demonstrated that a better service can be provided by the use of heterogeneous fleets

PREVIOUS WORK CONT'D…

Duran-Micco, Vermeir and Vansteenwegen (2020):

formulated a transit network design and frequency setting problem while considering a heterogeneous fleet

two objectives were considered: minimizing the total travel time and CO_2 emissions.

Results show that the heterogeneous fleet can reduce travel times and emissions simultaneously, compared to scenarios without a heterogeneous fleet.

CONTEXT

What issue are they solving?

Long waiting time

Passengers failing to board due to a lack of capacity

Bus capacity constraints

HOW DID THEY SOLVED IT?

✓Optimize dispatching patterns of a heterogeneous fleet of buses :

-Dispatching order

- Dispatching times

✓Model used:

-Mixed-Integer Nonlinear Programming (MINLP) model

✓Constraints used:

-Flow constraints(Eq.13–19)

-Vehicle movement constraints(Eq.7 –12).

-Resource availability constraints (Eq.4–6)

-Vehicle capacity constraints (Eq.16)

✓Mixed fleet used:

-12-meter (standard) bus: 70-passenger capacity

-15-meter (rigid) bus: 90-passenger capacity

18-meter (articulated) bus: 120-passenger capacity

PROBLEM FORMULATION

The mixed-fleet vehicle dispatching problem is formulated as follows:

(i)Waiting time for new passengers arriving at stops over the headway,

(ii) Extra waiting time for passengers who were unable to board the preceding vehicle due to a lack of capacity, who have to wait for the next vehicle(s).

- *For passengers in (i), the waiting time is averagely estimated as half of the headway (H/2), which is a well-known estimation in the literature on high-frequency bus services due to the random and unplanned arrival of passengers at stops*

- *for passengers in (ii), the extra waiting time is equal to the whole headway (H) because they have to wait for the next bus service.*

$$\underset{x_{mi}, T^d_{i,1}}{Min}\ \mathrm{AWT} = \frac{1}{P}\cdot \sum_{i\in V, i\geq 2} \sum_{j\in S} \sum_{k\in S, k>j} \left(\underbrace{\lambda_j \left[T^d_{i-1,j}\right].OD_{j,k}\left[T^d_{i-1,j}\right].H_{i,j}\cdot\frac{H_{i,j}}{2}}_{\text{(i)}} + \underbrace{N^f_{i-1,j,k}.H_{i,j}}_{\text{(ii)}} \right) \tag{1}$$

s.t:

$$x_{1i} + x_{2i} + x_{3i} = 1 \quad \forall i \in V \tag{2}$$

$$C_i = c_1 x_{1i} + c_2 x_{2i} + c_3 x_{3i} \quad \forall i \in V \tag{3}$$

$$\sum_{i\in V} x_{1i} = A \tag{4}$$

$$\sum_{i\in V} x_{2i} = B \tag{5}$$

$$\sum_{i\in V} x_{3i} = C \tag{6}$$

$$H_{i,j} = T^d_{i,j} - T^d_{i-1,j} \quad \forall i \in V - \{1\}, \forall j \in S \tag{7}$$

$$T^a_{i,j} = T^d_{i,j-1} + \delta_a + T^r_{i,j} + \delta_d \quad \forall i \in V, \forall j \in S - \{1\} \tag{8}$$

$$T^r_{i,j} \sim \mathrm{lognormal}(r_j, \sigma_j) \quad \forall i \in V, \forall j \in S - \{1\} \tag{9}$$

PROBLEM FORMULATION CONT'D..

$$T^d_{i,j} = T^a_{i,j} + T^s_{i,j} \quad \forall i \in V, \forall j \in S - \{1\} \tag{10}$$

$$h_{\min} \leq T^d_{i,1} - T^d_{i-1,1} \leq h_{\max} \quad \forall i \in V - \{1\} \tag{11}$$

$$T^s_{i,j} = \alpha_0 + P^a_i \alpha_1 N^a_{i,j} + P^b_i \alpha_2 N^b_{i,j} \quad \forall i \in V, \forall j \in S \tag{12}$$

$$N^w_{i,j,k} = \lambda_j \left[T^d_{i-1,j}\right] . OD_{j,k}\left[T^d_{i-1,j}\right] . (\overbrace{T^d_{i,j} - T^d_{i-1,j}}^{H_{i,j}}) + N^f_{i-1,j,k} \quad \forall i \in V - \{1\}, \forall j, k \in S, \quad j < k \tag{13}$$

$$N^w_{i,j} = \sum_{k \in S, k > j} N^w_{i,j,k} \quad \forall i \in V, \forall j \in S \tag{14}$$

$$N^{on}_{i,j} = N^{on}_{i,j-1} - N^a_{i,j-1} + N^b_{i,j-1} \quad \forall i \in V, \forall j \in S - \{1\} \tag{15}$$

$$N^b_{i,j} = \min\left\{N^w_{i,j}, C_i - N^{on}_{i,j} + N^a_{i,j}\right\} \quad \forall i \in V, \forall j \in S \tag{16}$$

$$N^a_{i,j} = \sum_{j' \in S, j' < j} N^s_{i,j',j} \quad \forall i \in V, \forall j \in S - \{1\} \tag{17}$$

$$N^s_{i,j,k} = \frac{N^b_{i,j}}{N^w_{i,j}} N^w_{i,j,k} \quad \forall i \in V, \forall j, k \in S, \quad j < k \tag{18}$$

$$N^f_{i,j,k} = N^w_{i,j,k} - N^s_{i,j,k} \quad \forall i \in V, \forall j, k \in S, \quad j < k \tag{19}$$

$$T^d_{i,1} \geq 0 \quad \forall i \in V \tag{20}$$

$$x_{mi} \in \{0, 1\} \quad \forall m \in M, \forall i \in V \tag{21}$$

As can be seen in expression (1), the objective of the problem is to minimize the average waiting time (AWT), obtained through

- Eq. (16) indicates that the number of passengers who can successfully get on bus i at stop j cannot be larger than the remaining capacity inside bus i at that stop.
- Eq. (19) enables us to compute the actual number of passengers being left

PROBLEM FORMULATION CONT'D..

The following is the list of notations

Table 1
List of notations.[5]

Symbol	Description	Unit
Sets		
V	Set of vehicles, $V = \{1,\ 2, \ldots, N_v\}$	
M	Set of vehicle types, $M = \{1, 2, 3\}$ for three available bus sizes	
S	Set of stops, $S = \{1,\ 2, \ldots, N_s\}$	
Indices		
i	Index of buses	
m	Index of bus type	
j, k	Index of stops	
Input parameters		
N_v	Total number of buses in a given fleet of mixed bus sizes ($N_v = A + B + C$)	
A	Available number of 12-m long buses in the given mixed fleet	
B	Available number of 15-m long buses in the given mixed fleet	
C	Available number of 18-m long buses in the given mixed fleet	
N_s	Number of stops along the bus route	
$\lambda_j[t]$	Passenger arrival rate at stop j at time t	pax/min
$OD_{j,k}[t]$	O-D matrix (the percentage of waiting passengers at stop j, who aim to travel from stop j to destination stop k) at time t	%
P	Total demand (the total number of waiting passengers who arrived at stops), which is a constant value during the entire study period	pax
δ_a	Acceleration time	s
δ_d	Deceleration time	s
r_j	Mean running times between stops $j-1$ and j	min
σ_j	Standard deviation of running times between stops $j-1$ and j	min
α_0	Time required for opening and closing bus doors	s
α_1	Average alighting time per passenger	s/pax
α_2	Average boarding time per passenger	s/pax
P_i^a	Proportion of passengers alighting through the busiest door of bus i	%
P_i^b	Proportion of passengers boarding through the busiest door of bus i	%
T_1	First dispatching time (beginning of the planning horizon)	min
T_2	Last dispatching time (end of the planning horizon)	min
$h_{\min}$	Minimum dispatching headway	min
$h_{\max}$	Maximum dispatching headway	min
c_1	Capacity of a 12-m long bus	pax/veh
c_2	Capacity of a 15-m long bus	pax/veh
c_3	Capacity of an 18-m long bus	pax/veh

PROBLEM FORMULATION CONT'D..

The following is the list of notations

Auxiliary variables		
C_i	Total capacity of bus i	pax/veh
$T^a_{i,j}$	Arrival time of bus i at stop j	min
$T^d_{i,j}$	Departure time of bus i from stop j, $j \geq 2$	min
$T^s_{i,j}$	Dwell time of bus i at stop j	min
$T^r_{i,j}$	Running time of bus i between stops $j-1$ and j	min
$H_{i,j}$	Headway between buses $i-1$ and i at stop j	min
$N^w_{i,j,k}$	Number of passengers with trip $j \rightarrow k$ waiting for bus i at stop j	pax
$N^w_{i,j}$	Total number of passengers waiting for bus i at stop j	pax
$N^{on}_{i,j}$	Number of passengers on bus i between stops $j-1$ and j	pax
$N^b_{i,j}$	Number of passengers boarding bus i at stop j	pax
$N^a_{i,j}$	Number of passengers alighting bus i at stop j	pax
$N^s_{i,j,k}$	Number of passengers with trip $j \rightarrow k$, who can board bus i at stop j	pax
$N^f_{i,j,k}$	Number of passengers with trip $j \rightarrow k$, who fail to board bus i at stop j	pax
Decision variables		
x_{mi}	Binary variable which is 1 if a type m vehicle is dispatched as i-th service; otherwise 0 ($m = 1, 2, 3$ for the three different bus)	
$T^d_{i,1}$	Dispatching time of bus i from the first stop	min

SOLUTION APPROACHES

✓Algorithm used:

- Simulated Annealing (SA) was used, which takes advantage of producing feasible neighboring solutions, to solve large real-world mixed-fleet vehicle dispatching problems within a reasonable computing time.
- In order to deal with travel time stochasticity, the SA algorithm is coupled with a Monte Carlo Simulation (MCS), and each solution is repeatedly assessed over several simulation-based evaluations through a Monte Carlo Simulation (MCS) method

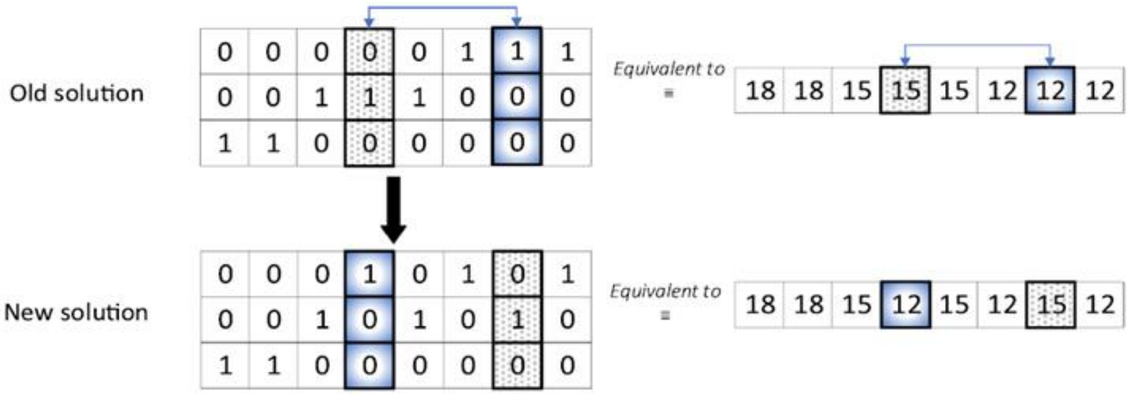
✓The proposed dispatching problem is a permutation-based problem in terms of vehicle dispatching order,i.e., a permutation of a given set of vehicles leads to a new ordering of those vehicles.

✓As an illustrative example of adjusting vehicle dispatching order, a set of 8 vehicles of different sizes was used: {12,12,12,15,15,15,18,18} can be dispatched in P(8 ;3 , 3 , 2) = $\frac{8!}{3!\times 3!\times 2!}$= 560 different arrangements.

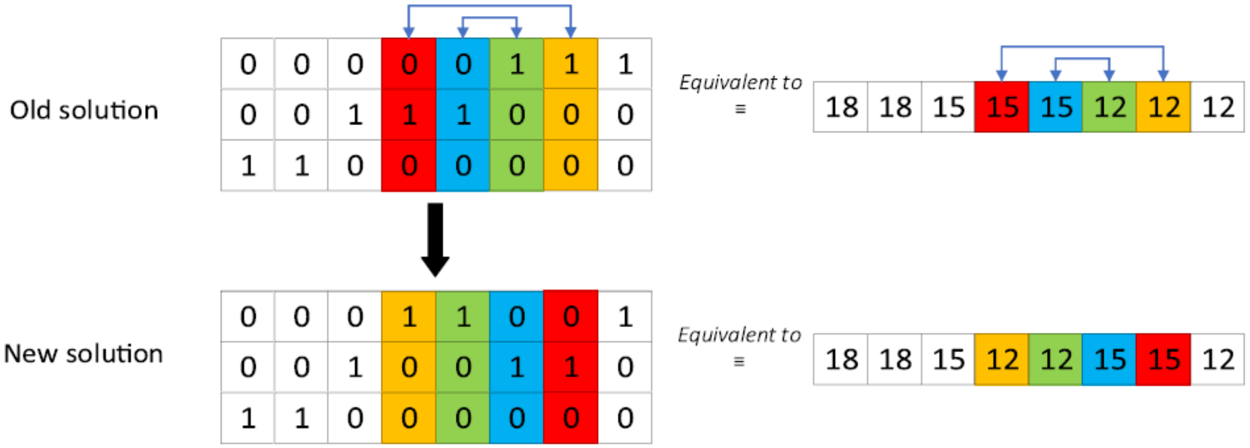
SOLUTION APPROACHES CONT'D..

✓ In the used SA algorithm, efficient operators were employed to produce diverse solutions in terms of dispatching sequence:

- Swap operator
- Inversion operator



(a) Swap operator



(b) Inversion operator

NUMERICAL EXPERIMENTS

Table 2
Computational results of the SA vs. GAMS in solving small and medium instances.

Class	Instance number	Instance features						Objective value			Comp. time (*sec*)	
		N_s	N_v	A	B	C	NLP*	GAMS	SA	GAP (%)	GAMS	SA
Small	#1	6	3	1	1	1	6	0.24	0.24	0.00	270	7
	#2	6	4	2	1	1	12	0.32	0.32	0.00	1450	14
	#3	6	4	1	2	1	12	0.30	0.30	0.00	1461	13
	#4	6	4	1	1	2	12	0.29	0.29	0.00	1455	14
	#5	8	5	1	1	3	20	0.36	0.36	0.00	3020	16
	#6	8	5	2	2	1	30	0.40	0.40	0.00	3002	15
	#7	8	5	2	1	2	30	0.39	0.39	0.00	3014	15
	#8	10	6	1	4	1	30	0.55	0.55	0.00	4681	17
	#9	10	6	1	1	4	30	0.49	0.49	0.00	4570	15
	#10	10	6	1	3	2	60	0.53	0.53	0.00	9365	17
	#11	10	6	1	2	3	60	0.51	0.51	0.00	9359	16
	#12	10	6	2	2	2	90	0.55	0.55	0.00	14,256	17
Medium	#13	12	7	5	1	1	42	0.94	0.94	0.00	7560	21
	#14	12	7	1	1	5	42	0.77	0.77	0.00	7551	19
	#15	12	7	4	2	1	105	0.92	0.92	0.00	18,910	20
	#16	12	7	4	1	2	105	0.88	0.88	0.00	19,215	21
	#17	12	7	1	3	3	140	0.82	0.82	0.00	25,536	22
	#18	12	7	2	2	3	210	0.85	0.85	0.00	38,316	20
	#19	14	8	6	1	1	56	1.56	1.56	0.00	13,448	25
	#20	14	8	1	1	6	56	1.13	1.13	0.00	13,372	22
	#21	14	8	1	2	5	168	1.21	1.22	0.83	40,328	25
	#22	14	8	2	4	2	420	1.41	1.41	0.00	>86,400	26
	#23	14	8	2	2	4	420	1.29	1.29	0.00	>86,400	24
	#24	14	8	3	3	2	560	1.45	1.46	0.69	>86,400	25
	#25	14	8	2	3	3	560	1.35	1.35	0.00	>86,400	24
	Max. gap%									**0.83**		

* No. of continuous NLP subproblems (i.e., $\frac{N_v!}{A! \times B! \times C!}$) solved by GAMS.

$$\mathrm{GAP} = \frac{(\mathrm{SA_{answer}} - \mathrm{GAMS_{answer}})}{\mathrm{GAMS_{answer}}} \times 100 \qquad (24)$$

This shows that the capability of SA's operators with their special neighbourhood search mechanisms is quite promising, as the designed swapping and inversion operators can fruit- fully generate a new feasible dispatching order of vehicles through a random displacement of vehicles within the same fleet, thereby enabling the algorithm for better exploitation of the best solutions in the feasible search space

4.1. Small and medium-sized test instances

- ✓ To obtain certain insights about the quality of the solutions found by the SA algorithm, the performance of the SA is evaluated by comparing its results to the optimal solutions obtained through the GAMS software 24.7.1 in solving a set of test problems generated randomly.
- ✓ 25 small and medium-sized test problems are randomly prepared with various sizes and features (see Table 2)
- ✓ The gaps between the best solutions found by the SA algorithm and the optimal solutions obtained by GAMS are computed using Eq. (24)
- ✓ The dispatching orders found by the SA are exactly the same as those obtained in the optimal solutions.
- ✓ The only difference is an insignificant gap (0.83%) in some dispatching times suggested by the SA compared to the optimal results of GAMS

NUMERICAL EXPERIMENTS CONT'D..

4.2. Application area and real-life case study (large-scale instance)

✓Military Road in North Sydney, Australia was used as a case study:

- It consists of a total of N_s = 24 stops (12 stops in each direction);
- Planning horizon from 7:00 am to 8:30 am was used

✓In the base case scenario, the following assumptions were made：

- The bus route is served by a given mixed fleet of 16 buses: {12, 12, 12, 12, 12, 12, 12, 12, 12, 15, 15, 15, 15, 18, 18, 18};

- Even dispatching headways of 6 minutes (service frequency of 10 bus/h);

- Constant passenger arrival rates at bus stops(arrival rates ($\lambda_j[t]$) are assumed to be constant during each 15-minute time interval(high-resolution demand)

-No passengers left behind (i.e. if buses never run at full capacity);

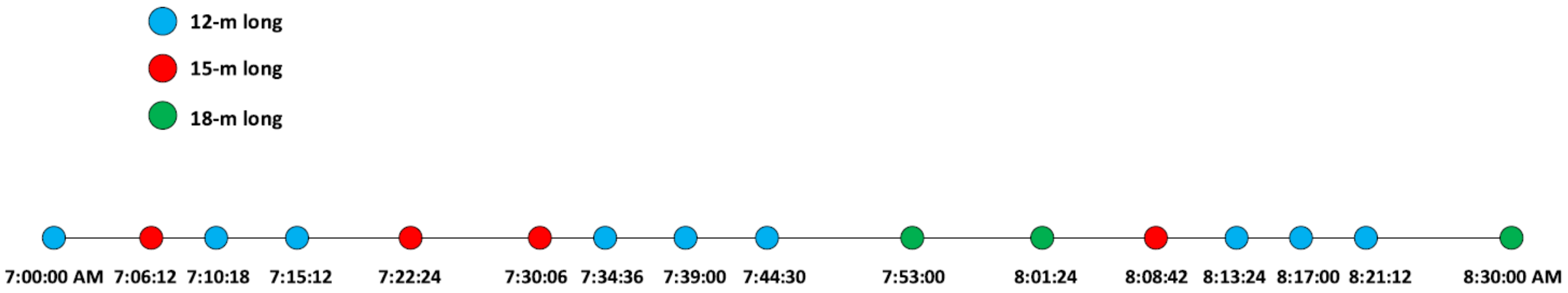
-The average waiting time would be 3 minutes.

NUMERICAL EXPERIMENTS CONT'D..

4.3. Optimal dispatching policy under high-resolution demand volumes

✓As shown in Fig. (a), under the optimal dispatching strategy, the following observations were made:

- Passengers experience an average waiting time of 3.55 (min/pax)
- In total, 9.9% of passengers are left behind and need to wait for a second bus to board, which explains that the average waiting time is larger than 3 minutes
- buses of one size are not necessarily dispatched consecutively one after the other



(a)

NUMERICAL EXPERIMENTS CONT'D..

4.4. Comparison to even headway solutions

✓The optimal solution from Section 4.3 was compared to the case in which buses are dispatched at a uniform headway of 6 minutes

✓Two even-headway dispatching scenarios were developed:

- (i) Same dispatching order as in Fig. (a), under the constraint of a fixed 6-minute dispatching headway [see Fig. b].
- (ii) Optimal dispatching order, under the constraint of a fixed 6- minute dispatching headway [see Fig. c].

✓ The following observations were made:

- In case (i) :

  - Buses in the optimal solution are dispatched at an even headway of 6 minutes while maintaining their dispatching order.

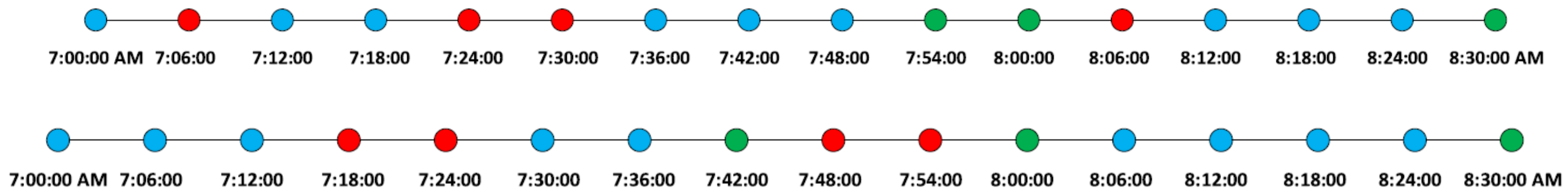
  - The number of passengers left behind, and consequently the average passenger waiting time increased by 55% and 11.5%, going from 309 to 480 (pax) and from 3.55 to 3.96 (min/pax) respectively

- In case (ii) :

  -It is assumed that vehicles are operated with a fixed 6-minute dispatching headway and only the dispatching order of each vehicle is optimized

  -The percentage of passengers left behind is 14% and passenger waiting time increases by 9%, reaching 3.87 (min/pax)

- This shows the benefits of dispatching buses at uneven headways in a situation with different bus sizes and binding vehicle capacity constraints.



(c)

NUMERICAL EXPERIMENTS CONT'D..

4.5. Comparing the optimal dispatching order with other predefined orders

- ✓Comparisons between the optimal solution and alternative dispatching schemes were conducted
- ✓Six different dispatching scenarios: D 12–15–18, D 12–18–15, D 15–12–18, D 15–18–12, D 18–12–15, and D 18–15–12 were tested in which buses are dispatched with a predetermined order and only the dispatching time of each bus is optimized
- ✓The following observations were made:

- As shown in the table, Overall, the average passenger waiting time increases broadly in line with the percentage of passengers left behind.
- In the optimal scenario, the average passenger waiting time is 3.55 (min/pax), followed by a value of 4.04 (min/pax) in scenario D 15–12–18
- By comparing the optimal scenario and scenario D 15–12–18, the optimal scenario leads to a decrease of 12.1% in the average passenger waiting, mainly caused by a further reduction in the percentage of passengers left behind, declining from 15.9% to 9.9%
- Using the optimal dispatching pattern instead of scenarios D 18–12–15 , D 12–15–18 , D 15–18–12 , D 18–15–12, and D 12–18–15 can produce savings in the average passenger waiting time by 25.7, 20.2, 19.7, 16.8, and 14.3 percent, respectively

Comparing the optimal dispatching order with other predefined orders.

Scenario	The average passenger waiting time (min/pax)	Percentage of passengers left behind (%)
SA solution	3.55	9.9
$D_{12-15-18}$	4.45	20.6
$D_{12-18-15}$	4.14	17.1
$D_{15-12-18}$	4.04	15.9
$D_{15-18-12}$	4.42	19.9
$D_{18-12-15}$	4.78	23.6
$D_{18-15-12}$	4.24	18.2

NUMERICAL EXPERIMENTS CONT'D..

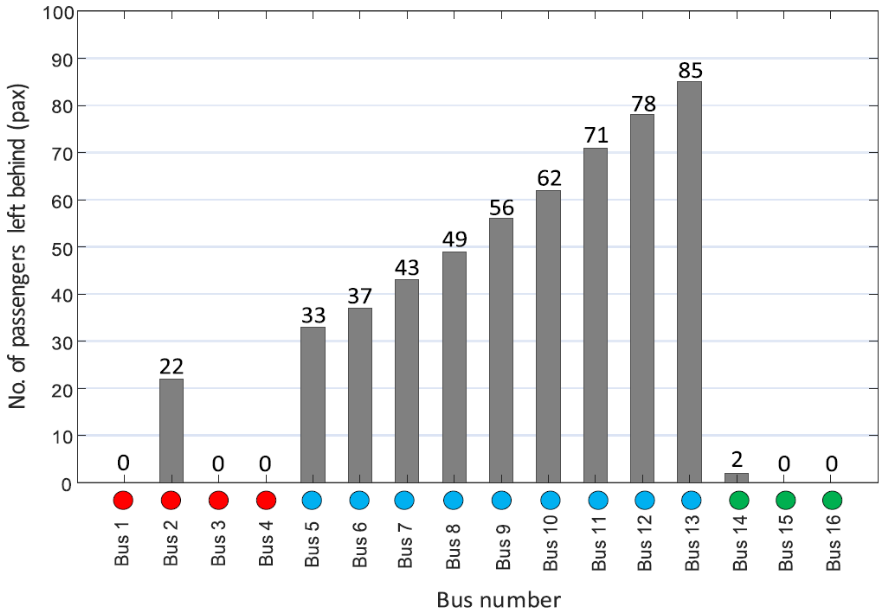
4.5. Comparing the optimal dispatching order with other predefined orders

✓The following figure gives information regarding the number of passengers left behind by each bus during the simulation time (a) in scenario D 15–12–18, and (b) in the optimal scenario.

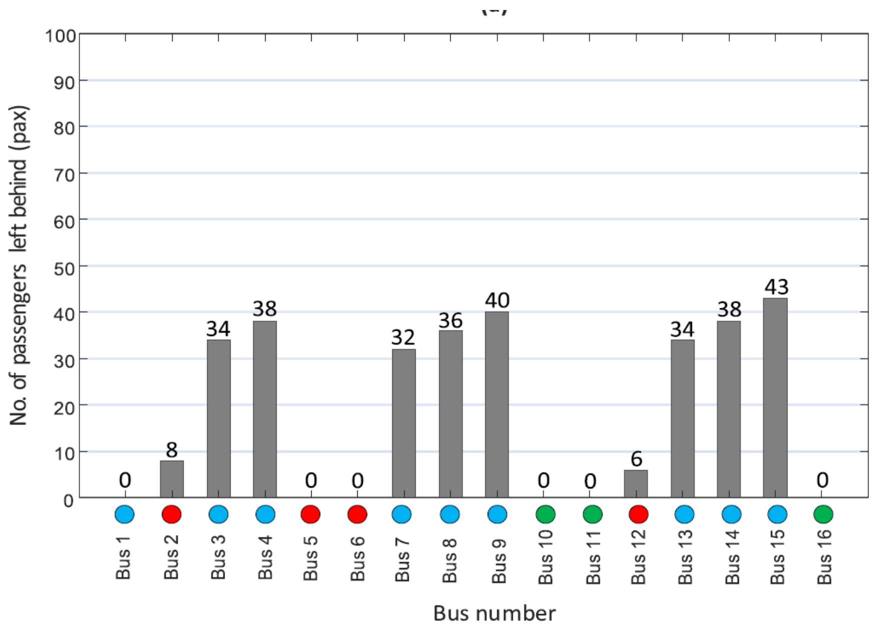
✓Looking firstly at the figure (a), we see that the number of passengers who fail to board increases steadily when 12-m long buses are dispatched sequentially.

✓Indeed, these buses do not have enough room to accommodate passengers who missed the previous buses due to a shortage of capacity, and consequently this situation will continue to deteriorate when they are dispatched sequentially.

✓As figure(b) shows, to optimize the capacity utilization of vehicles under the optimal scenario, buses of different capacities can be properly dispatched at specific times in accordance with demand conditions, and therefore the total number of passengers left behind by 12-m long buses reduces dramatically, dropping from 514 to 295 (pax).



(a)



(b)

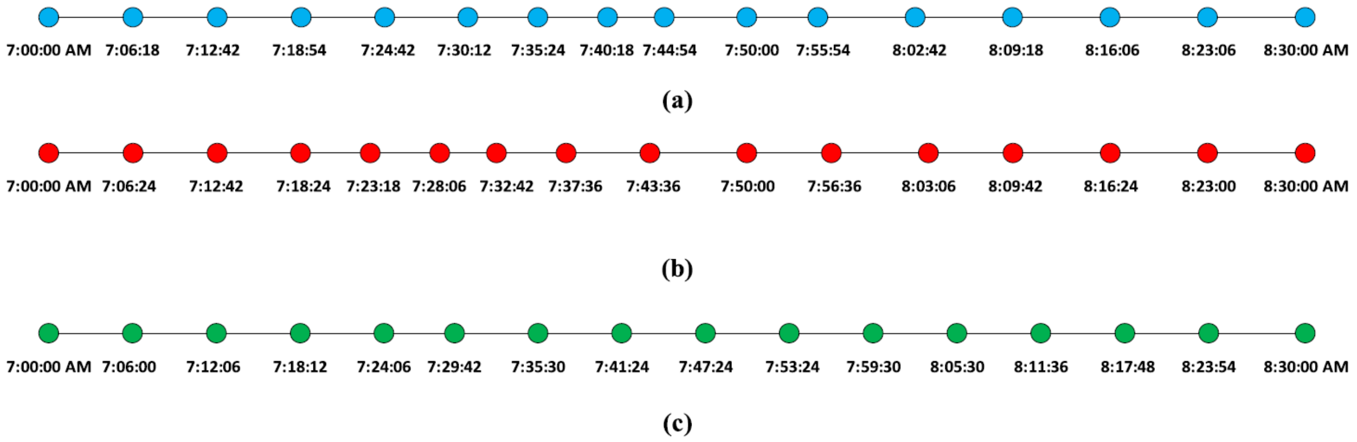
NUMERICAL EXPERIMENTS CONT'D..

4.7. Uniform fleet

✓A fleet with uniform bus sizes is available and only the dispatching time of each bus is optimized under high-resolution demand

✓The following observations were made:

- In uniform fleets with 12, 15, and 18-meter-long buses, the average passenger waiting time reaches the values of 5.96, 3.21, and 2.93 (min/pax) respectively,
- The percentage of passengers left be- hind is equal to 34.2%, 5.6%, and 0% respectively in these three cases, showing that passengers are not confronted with a lack of capacity when an 18-meter fleet is used.
- buses are not dispatched at quite even headways in order to deal with passenger demand fluctuation during the time of operation if capacity constraints are binding.
- This is clear for the case of 12-meter and 15-meter long buses, in which it is optimal to dispatch vehicles at uneven headways with coefficients of variation of 0.14 and 0.13, respectively ;
- On the other hand, for 18-meter-long buses, the dispatching headways are almost uniform, with a coefficient of variation of only 0.03
- It is clear that the optimality of uneven dispatching headways stems from two elements: having a mixed fleet and having localized peaks on demand that make buses run full.



CONCLUSION

Bus dispatching strategy has a profound impact on passenger waiting times.

In some cities, bus agencies have to combine vehicles of different sizes due to resource limitations together with historical reasons, e.g., when different sizes of buses are purchased at different times through different contracts

The fundamental question that needs to be addressed is how to optimally deploy a given mixed fleet with buses of different sizes (capacities) to provide services that minimize passenger waiting time.

A novel heterogeneous fleet dispatching problem as a Mixed-Integer Nonlinear Programming (MINLP) model was formulated to optimize vehicle dispatching schemes (in terms of dispatching order and dispatching time) in the case of time-dependent demand volumes.

Moreover, a Monte Carlo simulation framework is implanted as a subroutine into the SA to handle travel time uncertainty in the presence of stochastic travel times between stops.

To evaluate the effectiveness of the proposed model and the solution algorithm, a series of numerical experiments were conducted based on data from a real bus corridor under high-resolution demand volumes (15-minute-dependent demand volumes).

The results showed that, in addition to bus dispatching headway, bus dispatching sequence can strongly affect passenger waiting time in the mixed-fleet operation.

Moreover, to highlight the importance of the bus dispatching order, we also tested six different dispatching scenarios, in which buses of one size were always dispatched consecutively one after the other.

By comparing the optimal dispatching order with the worst-case scenario, we saw that the percentage of passengers left behind declined markedly from a peak of 23.6% to 9.9%, and consequently the average passenger waiting time went down by 25.7% to 3.55 (min/pax).

We found that the desirability of programming uneven bus headways depends on two factors: the existence of a fleet of vehicles of different sizes and of binding capacity constraints.

THANK YOU FOR YOUR KIND ATTENTION!